mining the fuel concentration and the total error of determining the model parameters;  $1/\phi$ , mean relative diameter of a hot particle. Dimensionless numbers:  $Nu = \alpha d_i/h$ ;  $Sh = \beta d_i/D$ ;  $Fr = (U-U_0)^2/gH$ ;  $Sc = \nu/D$ ;  $Ar = \frac{gd_i^3}{\nu^2} \frac{\rho_i - \rho}{\rho}$ .

### LITERATURE CITED

- 1. E. P. Volkov, M. N. Egai, and R. Yu. Shakaryan, Inzh.-Fiz. Zh., <u>52</u>, No. 2, 956-965 (1987).
- 2. V. A. Kolibabchuk and V. N. Orlik, Promysh. Teploenergetika, 7, No. 1, 85-88 (1985).
- 3. V. I. Dikalenko, Problems of Heat and Mass Transfer in Boiler Plant and Chemical Reactors [in Russian], Minsk (1983), pp. 68-77.
- 4. A. I. Tamarin, Inzh.-Fiz. Zh., <u>50</u>, No. 2, 260-266 (1986).
- 5. A. I. Tamarin and L. I. Levental, Inzh.-Fiz. Zh., 58, No. 4, 618-623 (1990).
- 6. V. V. Pomerantsev, K. M. Aref'ev, D. B. Akhmedov, et al., Fundamentals of Practical Combustion Theory [in Russian], Leningrad (1986).
- 7. L. I. Khitrin, Physics of Combustion and Explosions [in Russian], Moscow (1957).
- 8. G. I. Pal'chenok, and A. I. Tamarin, Inzh.-Fiz. Zh., 45, No. 3, 425-433 (1983).
- 9. G. I. Pal'chenok, and A. I. Tamarin, Inzh.-Fiz. Zh., 47, No. 2, 235-242 (1984).
- 10. A. I. Tamarin and Yu. S. Teplitskii, Inzh.-Fiz. Zh., <u>32</u>, No. 3, 469-473 (1977).
- 11. A. I. Tamarin and Yu. E. Livshits, Inzh.-Fiz. Zh., 39, No. 4, 19-25 (1980).
- A. I. Tamarin and Yu. E. Livshits, Vestsi Akad. Nauk BSSR, Ser. Fiz.-Energ. Nauk, No. 3, 129-130 (1977).
- A. I. Tamarin and Yu. S. Teplitskii, Vestsi Akad. Nauk BSSR, Ser. Fiz.-Energ. Nauk, No. 1, 88-94 (1977).
- A. Adzheyak, A. I. Tamarin, and K. E. Goryunov, Jnzynieria Chemiczna i Procesowa, <u>4</u>, No. 1, 45-52 (1983).
- A. I. Tamarin, Yu. E. Livshits, D. M. Galershtein, et al., Problems of Heat and Mass Transfer in Heat Energy Equipment, Gas Generators and Chemical Reactors [in Russian], Minsk (1985), pp. 154-164.
- A. I. Tamarin, L. I. Levental', D. M. Galershtein, et al., Problems of Heat and Mass Transfer in Heat Energy Installations with Disperse Systems [in Russian], Minsk (1985) pp. 37-42.
- 17. Rep. NPO TsKTI No. 062304/0-11703 [in Russian], Leningrad (1984).

# CHOICE AND OPTIMIZATION OF THE PARAMETERS OF A POROUS-SUBLIMATION COOLER

S. M. Ostroumov

UDC 536.422

The parameters of a porous-sublimation cooler (PSC) such that the temperature of the cooling surface a) does not exceed a prescribed value, b) is minimum, and c) does not exceed a prescribed value with minimum mass of the PSC are determined.

In the process of porous-sublimation cooling [1-3] heat is removed from the surface being cooled (Fig. 1) along a porous framework and is transferred to a solid refrigerant residing in the pores of the framework; this process results in sublimation of the refrigerant. Compared with other methods of sublimation cooling [4] a PSC substantially simplifies the construction of the cryosublimation systems, increases their reliability, and significantly improves heat transfer between the object being cooled and the refrigerant [2, 5]. When cryogenic systems are cooled it is usually necessary to choose the parameters of the PSC so that the temperature of the surface being cooled is minimum or does not exceed a prescribed value. For space cryogenic technology it is especially important to provide these conditions with minimum mass of the PSC.

Physicotechnical Institute of Low Temperatures, Academy of Sciences of the Ukrainian SSR, Khar'kov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 60, No. 6, pp. 918-922, June, 1991. Original article submitted May 3, 1990.

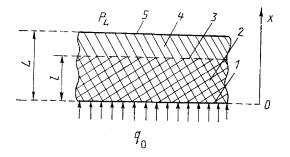


Fig. 1. Diagram of a porous-sublimation cooler: 1) the surface being cooled; 2) region containing the solid refrigerant (SR); 3) sublimation front of SR; 4) region not containing SR; 5) surface from which the vapor of the SR is evacuated.

<u>1. Choice of Parameters of PSC with an Irregular Porous Structure</u>. Let the heat flux density  $q_0$  be given on the cooled surface of the PSC (Fig. 1) and let the pressure  $P_L$  on the vapor evacuation surface be known. We are required to choose the parameters L,  $\lambda_c$ ,  $k_c$ , and  $\varepsilon_c$  so that over the time  $\tau$  the temperature  $T_0$  of the surface being cooled does not exceed a prescribed value  $T_m$ .

We shall assume that  $q_0$  and  $P_L$  are constant. Then we have [2]

$$T_0 = T_0(l) = T_l(l) + q_0 l / \lambda_c, \tag{1}$$

$$T_{1}(l) = 2\omega \ln^{-1} \left[ \delta^{2} / W(l) \right], \quad W(l) = P_{L}^{2} + 2q_{0} \Psi_{L}(L-l) / (\omega k_{c}), \tag{2}$$

$$l = l(t) = L - q_0 t / (\rho_1 \varepsilon_c \varepsilon_T \gamma), \tag{3}$$

where  $\omega = \gamma \mu / R_{g}$ ;  $\Psi_L = \Psi(T_L)$ ;  $\Psi(T) = T \eta_2(T)$ ;  $T_L = T_s(P_L)$ ;  $\delta = \text{const}$  is the parameter in the relation [4]

$$P_s(T) = \delta \exp\left(-\omega/T\right). \tag{4}$$

The expressions (1)-(3) are valid, if the following condition for the sublimation front to remain flat during the process of porous-sublimation cooling is satisfied [1, 3]:

$$P_s^2(T_l) \leqslant \varphi_L \lambda_c / k_c, \text{ where } \varphi_L = \varphi(T_L) \approx \varphi(T_l), \ \varphi(T) = \Psi(T) (T/\omega)^2.$$
(5)

Taking into account the fact that  $l(\tau) = 0$ , from Eq. (3) we obtain

$$L = q_{\rm b} \tau / (\rho_1 \varepsilon_{\rm c} \varepsilon_{\rm T} \gamma). \tag{6}$$

Since  $P'_{s}(T)' > 0$ , in order for the condition (5) to be satisfied it is necessary and sufficient that it be satisfied for  $T_{\ell} = \max T_{\ell}$ . From Eqs. (2), (3), and (6) we have

$$\max T_l = 2\omega \ln^{-1} \left( \frac{\delta^2}{P_L^2 + A/H} \right), \tag{7}$$

where  $A = 2q_0^2 \tau \Psi_L/(\omega \rho_1 \epsilon_T \gamma)$  and  $H = \epsilon_c k_c$ . Substituting  $T_\ell = \max T_\ell$  into Eq. (5) and taking into account (4) we find

$$\lambda_c \gg K_c \left( P_L^2 + A/H \right) / \varphi_L \,. \tag{8}$$

Let the relation (8) and therefore also the relation (5) be satisfied. Differentiating Eq. (1) with respect to l and taking into account Eqs. (2), (4), and (5) we obtain

$$T'_{0}(l) = [P_{s}^{2}(T_{l}) - \varphi_{L}\lambda_{c}/K_{c}]q_{0}/[\lambda_{c}P_{s}^{2}(T_{l})] \leq 0$$

Thus if the sublimation front remains flat, then the temperature of the surface being cooled increases as  $\ell$  decreases (as t increases), and vice versa. Thus, max  $T_0 = T_0|_{\ell=0} = T_{\ell}|_{\ell=0} = \max T_{\ell}$  (see Eqs. (1) and (2)), and from Eqs. (4) and (7) we find max  $T_0 \leq T_m$  with

$$H \geqslant A/(P_m^2 - P_L^2), \tag{9}$$

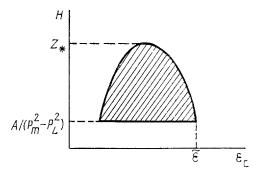


Fig. 2. The form of the region of values of  $H = \varepsilon_c k_c$  that satisfy the conditions (9) and (13).

where  $P_m = P_s(T_m)$ . Thus if the parameters L,  $\lambda_c$ ,  $k_c$ ,  $\varepsilon_c$  satisfy the relations (6), (8), and (9), then the temperature of the surface being cooled does not exceed the value  $T_m$ .

2. Optimization of the Parameters of Plate-Type and Tubular PSC. We shall study a platetype PSC with a framework consisting of parallel plates of thickness a. The spacing between the plates is equal to h; the surfaces of the plates are perpendicular to the surface being cooled. The parameters a and h must be chosen so that the flat sublimation front is stable and

2.1) the maximum (over the time  $\tau$ ) temperature of the cooled surface is minimum and

2.2) the mass of the PSC is minimum and the temperature of the cooled surface does not exceed the value  $T_m$ .

It is obvious that for plate-type PSC

$$\lambda_{\rm c} = (1 - \varepsilon_{\rm c})\lambda_3, \tag{10}$$

$$e_c = h/(a+h). \tag{11}$$

As estimates show [3], the flow of vapor between the plates of the PSC is usually laminar and in this case

$$k_{\rm c} = h^3 / [12(a+h)]. \tag{12}$$

We shall perform the optimization according to the criterion 2.1. Substituting Eq. (10) into Eq. (8), we obtain the condition under which the sublimation front remains flat in the form

$$H \leqslant Z(\varepsilon_{\rm c}), \tag{13}$$

where  $Z(\varepsilon_c) = [\varepsilon_c(1-\varepsilon_c)\lambda_3\phi_L-A]P_L^2$ . Since  $H \ge 0$  and the quadratic polynomial  $Z(\varepsilon_c)$  is nonnegative only for

$$\lambda_3 \geqslant 4A/\varphi_L,\tag{14}$$

the relation (14) is the necessary condition for the flat front to be stable. Let the condition of stability (13) be satisfied. Then max  $T_0 = \max T_\ell$  (see Sec. 1), and in addition max  $T_\ell$  decreases as H increases (see Eq. (7)). Therefore, in order to solve the optimization problem under the criterion 2.1 it is sufficient to find the parameters  $a = a_*$  and  $h = h_*$ for which H(a, h) is maximum under the condition (13). It is obvious that the equality in Eq. (13) corresponds to the parameters  $a_*$  and  $h_*$ , and the function  $Z(\varepsilon_c)$  at the point  $\varepsilon_c = \varepsilon_c^* = 0.5$  assumes its maximum value

$$Z_* = (\lambda_3 \varphi_L \, 4A) / (4P_I^2). \tag{15}$$

For this reason, taking into account Eqs. (11) and (12) we obtain

$$h_*/(a_*+h_*) = \varepsilon_c^* = 0.5, \ h_*^4/[12(a_*+h_*)^2] = \varepsilon_c^* k_c^* = Z_*.$$

Solving these equations for a\* and h\* we obtain

$$a_* = h_* = \sqrt{48Z_*}.$$
 (16)

1.00

Since max  $T_0 = \max T_{\ell}$ , substituting  $H = \max H = Z_*$  into (7) and taking into account (4) and (15) we find

$$\min\left(\max T_{0}\right) = T_{L} \left[1 - \frac{T_{L}}{2\omega} \ln\left(\frac{\lambda_{3}\varphi_{L}}{\lambda_{3}\varphi_{L} - 4A}\right)\right]^{-1}.$$
(17)

Thus the optimal, in accordance with the criterion 2.1, parameters of a plate-type PSC are given by the formula (16), and the minimum value of the highest (over the time  $\tau$ ) temperature of the surface being cooled is determined from Eq. (17). The parameters of a PSC with a tubular structure in the form of parallel cylindrical capillaries in a continuous matrix are optimized in an analogous manner. For this case the relations (10), (13)-(15), and (17) are also valid and, using instead of Eqs. (11) and (12) the relations  $\varepsilon_c = \pi R^2 n$ ,  $k_c = \pi R^4 n/8$  [3], we obtain

$$R_* = \sqrt{32Z_*}, \ n_* = 1/(2\pi R_*^2).$$

We now perform the optimization in accordance with the criterion 2.2. Since  $M_1 = q_0 \tau/\gamma = \text{const}$ , it is sufficient to minimize the mass  $M_c$ , which, taking into account Eq. (6), is equal to

$$M_{\rm c} = \rho_3 q_0 \tau \left(1 - \epsilon_{\rm c}\right) / \left(\epsilon_{\rm c} \rho_1 \epsilon_{\rm T} \gamma\right). \tag{18}$$

It is obvious that  $M_c$  decreases as  $\varepsilon_c$  increases, and in addition max  $T_0 \leq T_m$  under the condition (9), and the flat front is stable under the condition (13). For this reason, in order to optimize under the criterion 2.2 it is sufficient to find the highest value of  $\varepsilon_c$  for which the conditions (9) and (13) are satisfied. It is obvious that these conditions are satisfied only if max  $Z = Z_* \geq A/(P_m^2 - P_L^2)$ , whence taking into account Eq. (15) we find

$$\lambda_3 \ge 4AP_m^2 / [\varphi_L (P_m^2 - P_L^2)].$$
<sup>(19)</sup>

If Eq. (19) is satisfied, then  $Z(\epsilon_c) = A/(P_m^2 - P_L^2)$  has two positive, roots, the largest of which is

$$\tilde{\varepsilon} = \{\lambda_3 \varphi_L + [(\lambda_3 \varphi_L)^2 - 4\lambda_3 \varphi_L A P_m^2 / (P_m^2 - P_L^2)]^{0.5}\} / (2\lambda_3 \varphi_L)$$

For this case the range of values of H which satisfy the conditions (9) and (13) is shown in Fig. 2 (hatched region), whence one can see that the highest value of  $\varepsilon_c$  for which H belongs to the indicated region is  $\varepsilon_c = \tilde{\varepsilon}$ , and in addition  $Z(\tilde{\varepsilon}) = A/(P_m^2 - P_L^2) = H(\tilde{a}, \tilde{h})$ , where  $\tilde{a}$  and  $\tilde{h}$  are the optimal, under the criterion 2.2, values of a and h. Therefore, taking into account Eqs. (11) and (12), we obtain for a plate-type PSC

$$\tilde{h}/(\tilde{a}+\tilde{h})=\tilde{s}, \ \tilde{s}\tilde{h}^3=[12\ (\tilde{a}+\tilde{h})]=A/(P_m^2-P_L^2).$$

Solving this system for  $\tilde{a}$  and  $\tilde{h}$  we obtain

$$\tilde{h} = \sqrt{\frac{12A}{(P_m^2 - P_L^2)}} \tilde{\epsilon}, \ \tilde{a} = \tilde{h} (1 - \tilde{\epsilon})/\tilde{\epsilon}.$$

The minimum mass  $M_c$  is determined from Eq. (18) with  $\varepsilon_c = \tilde{\varepsilon}$ . The optimal parameters of PSC with a tubular structure are found in an analogous manner [3]:

$$\tilde{R} = \sqrt{\frac{8A}{(P_m^2 - P_L^2)}}/\tilde{\epsilon}, \ \tilde{n} = \tilde{\epsilon}/(\pi \tilde{R}^2).$$

## CONCLUSIONS

1. It was shown that under stationary external conditions, when the heat load and the pressure of evacuated vapors are fixed, a necessary and sufficient condition for the sublimation front to remain flat for a porous-sublimation cooling process is that the temperature of the surface being cooled must increase with time.

2. It was proved that the flat shape of the sublimation front in plate-type and tubular PCS can be stable only when the thermal conductivity of the material of the framework of the PSC is not less than a certain value (see Eq. (14)).

3. Relations were obtained for the parameters of PSC for which the temperature of the surface being cooled does not exceed a prescribed value and the sublimation front remains flat throughout the entire cooling process.

4. For plate-type and tubular PSC the parameters for which the flat sublimation front is stable and the temperature of the surface being cooled is minimum or does not exceed a prescribed value with minimum mass of PSC were determined.

# NOTATION

q, heat flux density;  $\lambda$ , thermal conductivity; k, permeability;  $\varepsilon$ , porosity;  $\varepsilon_T$ , fraction of the volume of pores filled with solid refrigerant; L, thickness of the PSC; &, coordinate of the sublimation front;  $\rho$ , density;  $\gamma$ , specific heat of sublimation;  $\mu$ , molecular weight of the refrigerant;  $R_g$ , universal gas constant;  $\eta$ , coefficient of dynamic viscosity; a and h, thickness of the plates and the spacing of the plates for a plate-type PSC; R and h, radius of the capillaries and the number of capillaries per unit area of the surface being cooled for a tubular PSC; M, mass per unit area of the surface being cooled;  $\tau$ , cooling time, P, pressure; T, temperature; and t, time. The indices are: o, surface being cooled; &, sublimation front; 1, solid refrigerant; 2, vapor; 3, material of the porous framework; L, surface from which vapor is evacuated; c, porous framework; s, state of saturation with the solid and gaseous phases of the refrigerant.

## LITERATURE CITED

- 1. S. M. Ostroumov, Inzh.-Fiz. Zh., 59, No. 6, 910-916 (1990).
- V. V. Druzhinets, N. M. Levchenko, and S. M. Ostroumov, Inzh.-Fiz. Zh., <u>60</u>, No. 5, 747-753 (1991).
- 3. S. M. Ostroumov, "Choice and optimization of the parameters of a porous-sublimation cooler," Preprint No. 25-89, Physicotechnical Institute of Low Temperatures of the Academy of Sciences of the Ukrainian SSR, Khar'kov (1989).
- 4. B. I. Verkin, V. F. Getmanets, and R. S. Mikhal'chenko, in: Thermophysics of Low-Temperature Sublimation Cooling [in Russian], Kiev (1980), pp. 115-122, 20-26.
- 5. V. V. Druzhinets and N. M. Levchenko, "Experimental investigation of the efficiency of porous structures for low-temperature sublimation cooling," Preprint No. 5-89, Physico-technical Institute of Low Temperatures of the Academy of Sciences of the Ukrainian SSR, Khar'kov (1989).